

Modeling Whale Populations

Jacob Biondo, Ethan Kerr, and Keelie Steiner

Abstract

Establishing a way to track whale populations presents a clear understanding as to how the population behaves and what needs to be done to guarantee the survival of the population. A logistic model with the Allee effect was utilized to perform mathematical calculations and generate graphical representations of the population. This model allowed for the possibility of extinction, providing accurate results for survival analysis (Amarti et al., 2017). Furthermore, the behavior was then analyzed to demonstrate how the population would increase or decrease depending on whether it was within the bounds of the minimum survival level, which is the number of population members required so breeding can happen, and the maximum environmental carrying capacity, which is the number of population members that can be supported by the environment, or whether it dropped below the minimum or above the maximum bound (Fadai et al., 2020). From this research, it is possible to gain an understanding of how to monitor whales to ensure their survival by using ordinary differential equation modeling techniques, such as a logistic model with the Allee effect.

When modeling whale populations, it is important to develop a model that will hold the population within the confines of the minimum survival level and the maximum environmental carrying capacity while accounting for the possibility of extinction. By doing so, the success and possible extinction of the population can be monitored (Amarti et al., 2017). In order to model this population, the logistic growth model with the Allee effect proves to be the best choice (Fadai et al., 2020).

Understanding the Results

Considering the survival of whales, the differential equation used to model the whale population is as follows:

$$\frac{dP}{dt} = k(M - P)(P - m)$$

P is the population, M is the maximum population of whales the environment can sustain, m is the minimum survival level, and k is a positive constant. $\frac{dP}{dt}$ represents the whale population at time t.

Through mathematical processes and simplifications, we generate the three cases of $P_0 < m$, $m < P_0 < M$, and $P_0 > M$, which can be solved, and the limit of population as time approaches infinity can be explained. For case one, $P_0 < m$ demonstrates that as the population decreases towards zero the whale population moves towards extinction as time goes to infinity. For case two, $m < P_0 < M$ demonstrates that the population increases towards the maximum environmental carrying capacity, M , as time approaches infinity. For case three, $P_0 > M$ demonstrates the population decreasing towards the maximum environmental carrying capacity, M , as time goes to infinity.

Another factor is the stability of the different cases. The equilibrium solution $P = m$, or the minimum survival level, is unstable. If $P < M$, the population decreases to 0; if $P > M$, the population increases to the carrying capacity, M . The equilibrium solution $P = M$ is asymptotically stable. If $P > M$, the population decreases towards M , and if $P < M$, the population increases towards M .

Implications and Conclusion

From the graphical analysis, it was found that if the population recedes beneath the minimum survival level, then the population will decrease towards zero until extinction is reached, or the population could bounce back. From the minimum survival level, the population will increase until it reaches the maximum environmental carrying capacity,

where it will approach the value of M . If the population exceeds the maximum environmental carrying capacity, then it will be forced to decrease to a value within the threshold of M .

Consequently, this project produced valuable results for marine biologists and ecologists to apply in order to understand the survival rate of the whale population. This leaves room for application to the entire ecosystem in terms of the dependence on whale populations and the effects this may cause for fisheries. Furthermore, pressing issues, such as climate change, are impacting the oceanic environment. It can be explored how climate change affects whale populations and what implications climate change can have on food supply and livability of the different oceanic areas. Expanding upon this project, the population of North Atlantic blue whales was analyzed and applied the model to understand the behavior of the population. Being an endangered species, it is necessary to monitor the population closely, and this model acts as a crucial tool in demonstrating the survival of the blue whales.

In all, this project stretches beyond the realm of only whale populations and allows for further scientific expansion on the importance of monitoring survival rates of populations as a whole while presenting several different impacts, decreasing or increasing, populations can have on an ecosystem. Similarly, these ideas stretch to other scientific concerns, such as climate change, and provide larger-scale applications to the results achieved.

References

- Amarti Z., Nurkholipah N. S., Anggriani N., Supriatna A. K., (2017). Numerical solution of a logistic growth model for a population with Allee effect considering fuzzy initial values and fuzzy parameters. *IOP Publishing*, 332, 1-2.
- Fadai N. T., Johnston, S. T., Simpson, M. J., (2020, September 16). Unpacking the Allee effect: Determining individual-level mechanisms that drive global population dynamics. *Royal Society*, 476(2241), 13-14.

Recommended Citation

Biondo, J., Kerr, E., & Steiner, K. (2023). Modeling whale populations. *Made in Millersville Journal*, 2023. Retrieved from <https://www.mimjournal.com/meteorology>